

Uncertainty In Auditing

Toshifumi Takada^{1,2}

Financial audits by Certified Public Accountants (CPA audits) have become mandatory for all listed companies, in accordance with legal requirements stipulated by Securities Law and the Companies Act in many countries. The audit regulatory requirement has expanded to other organizations such as universities, political parties, and labor unions. In addition to the financial audits, CPAs' duty have expanded to the Internal Control Report. The broadening scope of audit demonstrates that society requires public organizations and large companies to be transparent and to be responsible for the society.

KEYWORDS

Audit Judgment , Total Risk , Volatility , Uncertainty , Distribution, Fluctuation of Audit Judgment , Dual Audit Procedures , Audit Standards , Compliance , Internal Control

¹Emeritus Professor, Accounting School, Tohoku University, Japan

²Visiting Professor, Department of Accounting and Information Technology, National Chung Cheng University, Taiwan

Reference Format: T. Takada.(2023).Uncertainty In Auditing, International Journal of Computer Auditing, 5(1), 52-67. <https://doi.org/10.53106/256299802023120501005>

1 | BACKGROUND

A Certified Public Accountant (CPA) is an established professional specialized in accounting and auditing. When conducting audits, CPAs must comply with audit standards and follow the prescribed audit procedures. This sets them completely different from other professionals such as medical doctors and lawyers. However, it's essential to recognize that compliance with standards and rules doesn't mean that audit judgments are mechanical and uniform. Professional CPAs evaluate audit evidence and evidential matters, and make an independent and substantial judgment in accordance with the Audit standard requirement. This judgment is not formal and standardized; instead, it varies among individual CPAs.

Professional judgments differ from novice judgments because it is based on a profound and extensive knowledge, coupled with hands-on experience. Now, the crucial question emerges: What is the difference between the judgement made by a professional CPA and the one made by a novice? Is the degree of variance of a professional CPA's judgment smaller than that of novice person's judgment? Is there a possibility that the multiple professional CPA's judgments might be completely different among them?

This article focuses on the uncertainty of professional CPAs' judgments. Uncertainty, means the degree of confidence or the degree of ignorance of the CPA. In the field of stochastic theory, uncertainty is conceptually separated from volatility. My understanding of these two concepts are the same as stochastic theory.

2 | FOCUS OF THE ARTICLE

Conceptual Framework of Total Risk, Volatility, and Uncertainty

Let's begin by defining the concepts related to uncertainty. In our daily lives and business processes, it is required to tell the truth of an event. However, the nature of what is real or true is a philosophical question. Currently, we define truth as the conclusion based on all collected evidence related to the concerned event. However, it's crucial to note that the conclusion is not absolute in the sense that everyone anonymously reaches the single, same conclusion. The reason behind is that we cannot collect all of the evidence and evidential matters when we are required to get a conclusion in our daily living and doing business within limited time and resources. That means we cannot reach the absolute result about a past event or a future event because of economical limitations. Without these constraints, significant financial resources and time could be allocated to collect evidence and evidential matters and reach the nearly perfect truth. However, such a pursuit is impractical in our real society. For example, if a manufacturing company were to conduct destructive testing on all its products, it would be rendered incapable of selling any products.

In the realm of audit, truth is constantly exposed to total risk. Total risk is an amalgamation of volatility and uncertainty. While one can evaluate both volatility and uncertainty in a game with complete information, auditors operate in the practical realm of incomplete information. Consequently, auditors cannot reach the absolute truth and they have to express their tentative judgment as their opinions.

Case 1: Volatility = Known and Changing, Uncertainty = Calculated as Probability

In situations with complete information, confidence is understood as the theoretical degree of certainty in the inference of a result. For instance, if we are aware that a box contains six red balls and four white balls, our confidence in randomly selecting a red ball is 60%. Subsequently, if we do not return the red ball to the box, the confidence for the next selection being a red ball can be calculated as $(6-1) / (10-1) = 5 / 9 = 55.55\%$. In this simple example, the volatility (the quantity of each type of ball in the box) is known, and the confidence level changes with each trial, influenced by the outcomes of previous trials. The uncertainty is equivalent to the confidence level.

Case 2: Volatility = Fixed, Uncertainty = Inferred under the Normal Distribution

Consider a scenario where the ball is returned to the box after each selection, repeated ten times, with the color recorded at each trial. Replicate this process 10,000 times through computer simulation. It can be estimated that the selection of six red balls will be most frequent across all sets. We can also infer that zero red ball or 10 red ball will occur very few times. In this situation, the red ball selection depends on the initial condition in the box (six red balls and four white balls) and the number from 0 to 10 has the normal distribution. While the volatility (initial conditions in the box) is fixed and known, there is no 100% confidence in predicting the color of the next selected ball. Therefore, the result is uncertain. With an increasing number of trials, the distribution approaches a normal distribution, elevating the confidence level in proportion to the number of trials.

Case 3: Volatility = Fixed, Uncertainty = Inferred under the Normal Distribution

Imagine tossing a coin 10 times and repeating this process 10,000 times. Record the number of heads in each trial (ranging from 0 to 10). We have confidence that the recorded number of heads will mostly be around five, with occurrences of 0 or 10 heads being rare. The volatility (Heads or Tails) is known and fixed (50%), but the trial results exhibit a normal distribution. In that sense, the result is uncertain. The number of trials serves as evidence and evidential matter. As the number of trials increases, the distribution approximates a normal distribution, enhancing the confidence level in proportion to the trials conducted.

Incomplete Information and Judgment

In our daily lives or practices, we often operate within an environment of incomplete information, unable to precisely determine the volatility of an object or a event. Consequently, decisions and judgments must be made with uncertainty. However, we can estimate the distribution of occurrence of an event. In complete information scenarios, it is calculated as the probability value. On the other hand, in incomplete information scenarios, we just know the distribution and it fluctuates depending on the size of the population and the total number of trials. Uncertainty is thus defined as the estimation of this distribution.

Auditors operate in conditions of incomplete information, tasked with inferring the truth and estimating the degree of confidence to their final opinions, constrained by time and budget limitations. Their result (opinion) is based on the limited evidence and evidential matters so uncertainty cannot be avoided. The challenge lies in determining the confidence level, with the volatility (the client company's inherent risk) is not precisely understood (= changing), and uncertainty (the auditor's detection risk) depends on the amount of evidence and evidential matters. The uncertainty in audits depends on how deeply or how completely the auditors know the client company. This coincides with the degree of ignorance of the client on the side of auditors.

While auditors can control uncertainty, they cannot manage volatility, which depends on the client company. Recognizing the difference of these two concepts is crucial. When volatility is known and the degree of confidence is high, audit procedures doesn't need to be expanded. On the contrary, when volatility is unknown, and the confidence level is low, expanding procedures becomes essential. Subsequent chapters will present experiments focusing on the auditor's policy about the reasonable scope of audit procedure.

3 | SOLUTIONS AND RECOMMENDATIONS

3.1 | Experiment 1

Experiment Design

David Vose introduced an experiment demonstrating error about subjective inference. The same kind of experiment was conducted with participants from my undergraduate seminar class. Fourteen students attended this experiment, grouped into four groups consisting of two students each and two groups consisting of three students each.

Task: The participants were tasked with making interval estimations for the aggregated weight of all 14 students in the class. The goal was to create intervals as narrow as possible. Within each group, members were informed of the actual weight of their own group but were unaware of the real weights of the other groups.

Results

The outcomes of the experiment are presented in Table 1, showcasing the estimations made by each group. The actual aggregated weight was 927kg, with an average of 66kg and a standard deviation of 9.3kg.

Table 1: Results of Experiment 1

Group #	Number	Real	Average	Estimate	Estimated Interval	Result
1	3	83,63,74	73	66	60-71	correct
2	2	65,47	56	61	56-66	correct
3	2	65,47	56	59	54-64	out
4	2	63,65	64	60	57-64	out
5	2	75,58	67	62	57-63	out
6	3	90,64,68	74	62	58-66	correct
* Real average weight= 66kg						
** Real standard deviation= 9kg						
*** If the real weight 66kg is included the eaitimated interval,						
then the result is correct otherwise out.						

Discussion and Interpretation

One week after the experiment, I disclosed the actual weight of 927 kg to the students. Their task was to analyze and provide reasons why their estimates had fluctuated. The interval estimations made by each group was made by the appearance of other students. The fact that such subjective estimations had big fluctuations from the real weight 927 kg, as evidenced by the fact that only three groups' interval estimations included the actual weight.

The students discussed the reasons of fluctuations as follows::

1. They were unable to use the real weight data (evidence) within their own groups to infer the total aggregated weight.
2. They may have inferred the total weight more precisely if they could get more data from a few more groups.
3. Their confidence levels could have increased if they had access to different types of information, such as macro data on the weights of individuals of the same age.

3.2 | Experiment 2

Experiment Design

The United States boasts a history of over 50 years of research in bankruptcy forecasting models, with Professor Edward Altman pioneering this field. In this experiment, introduce an experiment using the bankruptcy forecasting model, where twelve Certified Public Accountants (CPAs) from an accounting firm cooperated.

Their tasks included:

1. Forecasting the likelihood of bankruptcy for a listed company based on its financial statements (balance sheet and income statement). The company was ranked the lowest among 500 listed companies by an economic journal⁴, utilizing Altman's model for this ranking.
2. Subsequent to task 1, four additional pieces of information were provided to the CPAs. Negative information included the bankruptcy of a key customer and the company initiating asset sales. Positive information included a key bank's decision to assist the company and another major company deciding to support it. Participants were then required to state whether or not they changed the previous forecasting.

Results

Table 2 displays the outcomes, where "0" refers to the forecast that the company will face bankruptcy within a year, and "1" refers to the forecast that the company will not face bankruptcy within a year. The column from 1st to 4th represents the change between "1" and "0" due to additional information. Table 3 provides a summary of the total number of changes observed between "1" and "0" based on the additional information provided.

Table 2: Results of Experiment 2

Table 2 Experiment: Fluctuation of Audit Judgment					
Subject	Initial	1st	2nd	3rd	4th
a	0	0	0	1	1
b	0	0	0	1	1
c	1	0	0	0	1
d	0	0	0	1	1
e	0	0	1	1	1
f	1	1	1	1	1
g	1	0	0	1	0
h	0	0	0	1	0
i	1	0	0	1	1
j	1	0	0	1	1
k	0	0	0	1	1
l	1	0	0	1	1
	*0: Bankruptcy, 1: Continue				
	**1st and 2nd: negative information added				
	***3rd and 4th: positive information added				

Table 3: Total Number of Changes based on the additional information provided

Table 3	Turnover from Initial Judgment											
Subject	a	b	c	d	e	f	g	h	i	j	k	l
Frequency	2	2	3	2	3	0	3	1	2	2	2	2

As shown in Table 2, the six CPAs forecasted "YES" and the same six forecasted "NO" initially. The five out of six examinees forecasting "YES" initially changed to "NO" by the positive information 3 and 4. All six examinees forecasting "NO" initially changed to "YES" by the negative information 1 and 2. On the other hand, two out of six examinees forecasting "NO" initially changed to "YES" by positive information 3 and 4. Also two out of six forecasting "YES" initially changed to "NO" by negative information 1 and 2.

Interpretation

Table 3 provides a summary of the changes observed in the initial forecasts from "YES" to "NO" and from "NO" to "YES" based on the additional information. The CPAs' judgments exhibited fluctuations, demonstrating that the additional information influenced the degree of confidence (uncertainty) in their initial assessments, prompting them to revise their judgments accordingly.

3.3 | Estimation of Uncertainty

Case 1: Events follow a normal distribution

When events follow a normal distribution, we can evaluate the uncertainty by using a distribution. For example, the distribution of the sample average weight of individuals within a specific age and sex group conforms to a normal distribution. Now let's consider a population average as μ , standard deviation as σ . And a sample average as \bar{x} , sample size as n , then

$$\mu = \bar{x} \pm \frac{\sigma}{\sqrt{n}}$$

is well known.

In the case of a normal distribution $N(\mu, \sigma^2)$, 68% of the total area is included within $\mu \pm \sigma$, and 95% is included within $\mu \pm 2\sigma$

Table 4 shows the estimation of μ from a sample group average. It hypothesizes that the sample average weight follows a normal distribution, allowing us to estimate a real value (truth) from the distribution. This hypothesis holds true when experiments are conducted numerous times. However, if the experiments are performed infrequently or only once, the result may assume an extreme value, deviating from a normal distribution.

Table 4: Interval for Each Group

Table 4 Interval in each group						
Group	Number	Weight	Interval (68%)	μ within interval	Interval (95%)	μ within interval
1	3	83,63,74	67-79	Out	60-86	In
2	2	65,47	47-65	Out	38-74	In
3	2	65,47	47-65	Out	38-74	In
4	2	63,65	55-73	In	46-82	In
5	2	75,68	58-76	In	49-85	In
6	3	90,64,68	68-80	In	61-87	In
Number of Respondents = 14						
Total Weight of 14 = 927kg						
Average Weight = 66kg						
Standard Deviation = 9kg						

In Experiment 1, the teams' expectations widely deviated from the actual value of 66 kg. Among them, three teams' expectations fell within the 95% interval ($\mu \pm 2\sigma$) of a normal distribution $N(\mu, \sigma^2)$. This interval is determined by the area covered, where $\mu \pm \sigma$ encompasses 68%, and $\mu \pm 2\sigma$ covers 95%, representing the degree of confidence = uncertainty. The required precision of the estimation depends on the degree of confidence. With a lower degree of confidence (e.g., 68%), the estimation interval can be broader. On the other hand, with a higher degree of confidence (e.g., 95%), the estimation interval must be narrower. In the former case, all six teams' estimations were within the interval, while in the latter case, only three teams' estimations were included.

Case 2: Standard deviation is unknown

In many audit cases, auditors don't know the population's standard deviation σ when they estimate the population's true value μ . In the traditional statistics, the estimation can be done using a t distribution as follows.

Consider:

sample average as \bar{x} , variance as s^2 , standard deviation as s ,

nonbiased population's average as $\hat{\mu}$, nonbiased population's variance as $\hat{\sigma}^2$,

nonbiased population's standard deviation as $\hat{\sigma}$,

sample size as n

The relationship $\hat{\sigma}^2 = \frac{n}{n-1} s^2$ is well-known.

Although it might be considered that estimating μ by using s^2 , supposing that $\hat{\mu} = \bar{x}$, is feasible, it is not possible because s^2 doesn't follow a normal distribution. Therefore, estimating μ by using \bar{x} and s^2 from a normal distribution is not applicable.

If

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1),$$

then we can estimate μ but this is not the case.

However, as $\hat{\sigma}$ is an unbiased value of σ ,

$$\hat{\sigma} = \sqrt{\frac{n}{n-1}} s$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{\bar{x} - \mu}{\sqrt{\frac{n}{n-1}} \frac{s}{\sqrt{n}}} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}}$$

$\frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}}$ follows a t distribution.

Therefore, when we don't know the standard deviation σ and need to estimate μ , we can use

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}}$$

Then,

$$\mu = \bar{x} \pm t \frac{s}{\sqrt{n-1}}, \text{ we can estimate } \mu.$$

When the degree of freedom is $\phi=1$, the degree of confidence of 90% is 6.314, and for 95%, it is 12.706. This estimation of μ with an interval represents the auditor's uncertainty. A t distribution is flatter than a normal distribution, indicating that as the sample size decreases, the interval becomes larger. If a higher confidence level is required, the interval becomes wider. In some cases, the interval may become too wide to have meaningful estimation. In Experiment 1, each group's sample size is $n=2$ or $n=3$. If the degree of confidence is 99%, the interval could range from 24 kg to 124 kg. Such an estimation may lack meaningful interpretation. As is well-known, in the case of a t distribution, the sample size should be $n \geq 30$ for the distribution to approximate a normal distribution.

Case 3: Discrete data and controlling uncertainty

Accounting data consists of continuous numbers, but auditors often deal with discrete data in their practice. For instance, when assessing business continuity, auditors must make YES/NO judgments at various points, as depicted in the flowchart outlined in Audit Standards. When an auditor makes a negative judgement, as a Yes/No judgement, on business continuity, additional procedures are required. After that, the final decision will be expressed in the audit report. In Experiment 2, all the auditors changed their initial decision when they got additional, positive information. But there is a possibility that their final decisions are opposite and conflict with each other.

Events with YES/NO outcomes (1/0 or red/white, etc.) are called discrete data. Audit judgments such as the business continuity are discrete data. The typical distribution of discrete data is a binomial distribution. Consider throwing a six-sided dice, where each outcome has an equal probability of $1/6$, forming a uniform distribution.

Imagine throwing 10 dice simultaneously and repeating this process 100 times. Count the occurrences of the face of a single pip. There will be 11 cases in each trial, from no occurrence to 10 times. The results follow a binomial distribution. Consider p as a probability of a specific outcomes ($1/6$) and q as a probability of non-specific outcomes ($5/6$). Then a binomial distribution is

$$P(r) = {}_n C_r p^r q^{n-r}$$

Here, C is the number of combinations. As the probability of dice is known, a binomial distribution represents uncertainty.

In Experiment 3, the initial number of YES/NO was 6/6. Individuals who initially said YES changed to NO after receiving additional positive information. This means that audit judgements will become uniform if auditors get additional information.

The number of audit reports with a negative judgment about business continuity is quite low. According to our survey⁹, the average number of audit reports with a negative opinion is almost 50 every year. The total number of listed companies is 3,500 and the probability is about 1.4. With such a low probability event, using a binomial distribution to calculate probability can be cumbersome. Therefore, a Poisson distribution is recommended:

$$P(r) = \frac{(np)^r e^{-np}}{r!}$$

Here, np is the average number of occurrences of a specific event in the trials (with a probability p , e.g., $1/6$ for a single dice).

We can apply a Poisson distribution to the audit of business continuity. Consider $n=100$, $p=1$, then $e^{-1}=0.368$. If the number of clients is 100, then

The probability of 1 client audit report being negative = $100 \times 0.01 \times 0.368 = 0.368$

The probability of 2 client audit report being negative = $\frac{1}{2} \times 0.368 = 0.184$

...

The probability of 5 client audit report being negative = $\frac{1}{120} \times 0.368 = 0.003$

These probabilities represent a volatility of audit judgement about business continuity. As Experiment 2 shows, the judgments of auditors are diversified initially, but with additional information, their judgments become more uniform. This means that when volatility is known, the diversification of audit judgments is attributed to insufficient information (evidence and evidential matter). By additional information, the uncertainty of audit judgments can be controlled.

4 | HOW TO CONTROL UNCERTAINTY IN CPA'S JUDGMENT

4.1 | Separation between Volatility and Uncertainty

Volatility is the condition of a system and is often quantified as the probability of an event occurring within the system. In some cases, we know the probability, such as coin tossing, throwing 10 dice, or audit judgements of business continuity. For instance, the probability of getting heads or tails when tossing a coin is 50/50. This probability doesn't fluctuate. Although the forecast of the next coin toss may fluctuate according to our minds, the confidence in the aggregate outcome of multiple tosses leans towards an even distribution of heads and tails. Our confidence level, or uncertainty, is influenced by our understanding of volatility.

If we know the distribution of uncertainty, we can get it from the distribution. However, if we don't know both volatility and uncertainty, how can we get to know our degree of confidence? This is the issue to separate volatility from uncertainty.

In real-world situations, auditors operate in a mixed environment with both volatility and uncertainty. However, we know that experienced practitioners often reach uniform conclusions based on similar evidence. This is a reasonable hypothesis, given that experienced practitioners' judgments tend to be stereotyped. In practice, an audit team is composed of an experienced CPA (partner) and middle and young CPAs. The volatility of the client's system can be known to an audit team by the knowledge of an experienced CPA. Then the issue to be solved is to know the uncertainty.

Consider that a junior CPA in an audit team is tasked with estimating the possibility of accounts receivables collection. Generally speaking, as a junior CPA may struggle to distinguish between volatility and uncertainty, his/her estimations are often based on total risk, where Total risk = volatility + uncertainty. We hypothesize that the estimation made by an experienced CPA (partner) represents volatility. As a result, we can get uncertainty = Total risk - volatility. The uncertainty may be diversified. We need to control such diversification. This implies that the result of the audit procedure done by junior CPAs must be monitored or checked by an experienced CPA.

4.2 | How Much the Auditor Should Gather Evidence

As uncertainty is defined by the degree of ignorance, auditors have the potential to control uncertainty by increasing the amount of evidence. The question then arises: How much evidence should auditors accumulate? In statistics, if the sample size becomes larger, the preciseness of the estimation of a parameter can be improved. Average of sample data follows a normal distribution as

$$N\left(\bar{x}, \frac{\sigma^2}{n}\right)$$

When estimating μ , the confidence interval can be defined as follows:

$$90\% \text{ confidence level: } \mu = \bar{x} \pm 1.65 \times \frac{\sigma}{\sqrt{n}}$$

$$95\% \text{ confidence level: } \mu = \bar{x} \pm 1.96 \times \frac{\sigma}{\sqrt{n}}$$

This implies that as the sample size increases from 2 to 10, the interval becomes narrower, from $\frac{1}{\sqrt{2}}$ (0.7) to $\frac{1}{\sqrt{10}}$ (0.3), effectively almost doubling the improvement¹⁰.

Now, let's consider a sample size of 100. The interval shrinks to 0.1, showing an improvement from 0.3 when the sample size was 10. In summary:

Sample size from 2 to 10 (difference of 8) → Improvement from 0.7 to 0.3

Sample size from 10 to 100 (difference of 90) → Improvement from 0.3 to 0.1

The sample size represents a cost and improvement represents a performance. We understand that the relationship between cost and performance is not proportional.

Economic theory suggests that the optimal sample size can be determined through a cost-benefit analysis. In some audit practices, too many procedures tend to be performed, but it's essential to understand that increasing audit procedures increases the cost but not necessarily performance. We need to escape from meaningless audit procedures.

4.3 | Significance of Audit Procedure Complied with Standards

There was a case that accounting judgments from two different major firms conflicted with each other¹¹. Their judgments were opposite, resulting in conflicting YES/NO conclusions. The root cause of this discrepancy was related to accounting standards. At that time, there was not an applicable audit standard. The auditor in charge faced significant pressure from the client, given the potential for opinion shopping, and the client preferred a clear opinion.

Nowadays, Audit Standards have been supplemented by Statements on Audit Standards provided by professional organizations such as AICPA and JICPA. These standards have become highly detailed. Consequently, Professional auditors will reach the same opinion from the same evidence as they have to comply with Audit Standards which have

become guidelines about how to evaluate audit evidence.

Audit judgments are composed of volatility and uncertainty. Volatility, caused by the client system, is a matter of fact finding. Auditors must collect evidence about the client system by audit procedures prescribed by Audit Standards. CPAs with enough experience and knowledge can reach the same conclusion from the same evidence because of their adherence to Audit Standards. In this context, we can understand that Audit Standards play an important role.

4.4 | Significance of Audit Team

A listed company is audited by an audit team, and the size of which depends on the client company's size. Typically, an audit team consists of two seniors, two managers, and one or two juniors, with additional technical staff supporting the team. In the case of a sizable client, the team's size may expand to 10, accompanied by a substantial technical staff.

Within an audit team, managers and juniors are assigned the responsibility of executing audit procedures. They are responsible for collecting and evaluating evidence for each proposition to be verified. However, the conclusions they reach are considered tentative because a senior has a duty to verify the tentative conclusion. This establishes a dual audit procedure for each proposition ¹².

The implementation of dual procedures aims to minimize human operational errors. For example, accounting requires double checking in many procedures. Similarly, separation of a job is traditionally required in cash transactions. A possible mistake is a kind of uncertainty as mistakes are thought to be diversification from a correct conclusion.

Even professional auditors cannot be 100% confident of their conclusions. The dual audit procedure serves to decrease the degree of uncertainty. Major audit firms have their own quality control systems to verify the audit procedures of every audit team. In addition, public oversight boards monitor audit firm procedures, creating dual and triple verification systems in various countries.

Quality is proportional to cost. While multiple verification mechanisms enhance audit quality, they also increase audit costs. Audit fees are determined through negotiation between audit firms and clients, based on the market principle of supply and demand. The adoption of dual audit procedures within audit teams is now a method adopted by audit firms to achieve the necessary audit quality while managing audit costs.

5 | FUTURE RESEARCH DIRECTIONS

5.1 | Principle of increasing evidence

Pick up a few samples from a normal distribution $N(\mu, \sigma^2)$. Consider the number of samples is 2, then the distribution of the addition of these samples follows a normal distribution with an average of 2μ and a standard deviation of $\sqrt{2}\sigma$. The distribution of the average of samples follows a normal distribution with an average of $2\mu \div 2 = \mu$ and a standard deviation of $\sqrt{2}\sigma \div 2 = \frac{\sigma}{\sqrt{2}}$. Consider the number of samples is n , then the standard deviation is $\frac{\sigma}{\sqrt{n}}$. If this standard

deviation is known, then we can estimate the population average as:

$$\mu = \bar{x} \pm \frac{\sigma}{\sqrt{n}}$$

Consider the sample size as n , then the sample variance s^2 is given by:

$$s^2 = \frac{\sum(x - \bar{x})^2}{n}$$

For $n = 2$, when estimating the population variance $\hat{\sigma}^2$, we need to subtract 1 from the total number 2 to calculate the average because the estimated average is given, and the other data is fixed. Therefore, we must subtract 1 from the number of data in the denominator. The subtracted number is called the degree of freedom ϕ . Therefore, $\phi = 2 - 1 = 1$.

When estimating the population variance, the degree of freedom is 1.

Therefore

$$\hat{\sigma}^2 = \frac{\sum(x - \bar{x})^2}{n - 1}$$

As $s^2 = \frac{\sum(x - \bar{x})^2}{n}$ can be substituted into the right side:

$$\hat{\sigma}^2 = \frac{n}{n - 1} \times \frac{\sum(x - \bar{x})^2}{n} = \frac{n}{n - 1} s^2$$

The t distribution table is a matrix of confidence level (P) and degrees of freedom (ϕ):

For $P = 0.1$, $\phi = 30$ then $t = 1.697$

For $P = 0.05$, $\phi = 30$ then $t = 2.042$

Considering the distance from an average as Z and the area as $I(Z)$ in $N(0,1)$

For $Z = 1.69$ then $I(Z) = 0.4545$

For $Z = 2.04$ then $I(Z) = 0.4793$

$I(Z)$ represents one side, and both sides' areas are 0.9090 and 0.9586. We can consider the confidence level as 1-area, then $P = 0.0910$, and $P = 0.0414$. These values are close to the t distribution table.

5.2 | Conclusion

Auditors are required to adopt risk-based audit procedures. This paper separate audit risk into audit volatility and audit uncertainty. Audit volatility is caused by the client system. On the other hand, audit uncertainty is caused by the degree of ignorance. Auditors can evaluate volatility by investigating a client's system. Although it can't be controlled by auditors but it can be evaluated by the collected audit evidence and evidential matter. On the other hand, audit uncertainty can be controlled because it is caused by an auditor.

The following is the conclusion of this paper:

(1) Quantity of Audit Evidence: Increasing the amount of audit evidence leads to a decrease in audit uncertainty. However, it is acknowledged that the cost-effectiveness of audit evidence diminishes as the volume of evidence increases.

(2) Significance of Audit Standards: The absence of Audit Standards results in substantial fluctuations in audit judgments. Even among experienced auditors, conflicting opinions may arise. Audit Standards serve as guidelines for practitioners, effectively reducing these fluctuations.

(3) Significance of Audit Teams: An audit team consisting of seniors, managers, and juniors is organized for a financial audit of a listed company. The implementation of a dual audit procedure for each proposition that needs to be verified helps control and reduce the degree of uncertainty in the audit process.

References

Journal Article

1. Altman, E.I. (1968). Financial Ratio, Discriminate analysis and prediction of corporate bankruptcy. *Journal of Finance, September*, 589-609.
2. Denne, J.S., & Christopher, J. (1999). Estimating the sample size for a t-test using an internal pilot, *Statistics in Medicine*, 18, 1575-1585.
3. Financial Service Agency, Japanese Government, Business Accounting Council. (2005). *Audit Standards*.
4. Financial Service Agency, Japanese Government, Business Accounting Council. (2005). *Quality Control Standards on Auditing*.
5. Hellerstein, D., & Mendelsohn, R. (1993). A theoretical foundation for count data model, *American Journal of Agricultural Economics*, 75, 3, 604-611.
6. Japanese Association of Certified Public Accountants. (2004). *Statement on Auditing Standards #22 Auditing Business Continuity*.
7. Stein, C.M. (1981). Estimation of the mean of a multivariate normal distribution, *The Annals of Mathematics*, 9, 6, 1135-1151.
8. Takada, T., & Oikawa, T. (2005). The states of GC audit in Japan, *Conference Proceedings of 17th Asian Pacific Conference on International Accounting Issues*. Wellington.
9. Takada, T., & Oikawa, T. (2006). Discretionary accruals of financially distressed companies – Ex-post analysis of default companies in Japan –, *Conference Proceedings of 18th Asian Pacific Conference on International Accounting Issues*, Maui.
10. Takada, T. (2007). *Foundations of audit risk*. Tokyo: Dobunkan.
11. Takada, T. (2008). *Auditing business continuity and bankruptcy forecasting model*. Tokyo: Dobunkan.
12. Yamaguchi, A. (2003). *Why Risona bank auditor suicided*. Tokyo: Nikkei Business.

Book

1. Hosoya, Y. (2004). *Statistical evidence and its interpretations*. Tokyo: Makino Shoten.
2. Tohoku University Statistics Group. (2002). *Statistics*. Tokyo: Yuhikaku.
3. Vose, D. (2008). *Risk analysis: A quantitative guide*. New Jersey: John Wiley & Sons.

Internet

1. Japanese Institute of Certified Public Accountants. (2004). *Statement on Auditing Standards 570 Auditing Business Continuity*. Retrieved December 22, 2022
https://jicpa.or.jp/specialized_field/570.html
2. Japanese Government. (2019). CPA Law. Retrieved December 22, 2022
<https://elaws.e-gov.go.jp/document?lawid=323AC0000000103>

Additional Readings

Book

1. Ijiri, Y. (1967). *The foundations of accounting measurement*. New York: Prentice-Hall International.
2. Ijiri, Y. (1975). *Theory of accounting measurement*. Florida: American Accounting Association.
3. Paton, W.A., & Littleton, A.C. (1940). *An introduction to corporate accounting standards*, Florida: American Accounting Association.
4. Shirata, Y. (2003). *Corporate bankruptcy forecasting model*. Tokyo: Chuokeizaisha.
5. Shirata, Y. (2019). *Bankruptcy forecasting model and corporate ranking by artificial intelligence*. Tokyo: Zeimukeiryokai.

Journal Article

1. Takada, T., Sakaki, M., Aoyagi, S., & Kawaguchi, H. (2019). Cybersecurity and AI – implications for internal auditing –, *Computer Audit*, 40, 31-48.
2. Takada, T. (2020). Study of CIM and IoT – Simulation for cost performance analysis –, *Computer Audit*, 42, 4-25.
3. Takada, T., & Suga, Y. (2021). School accounting education in Japan– In relation to economic development of the country –, *Computer Audit*, 44, 62-74.
4. Takada, T. (2022). Volatility of semiconductor companies, *Encyclopedia of Data Science and Machine Learning*, IGI Global.

Key Terms and Definitions

Audit Judgment: Certified Public Accountants are professionals in auditing. They have enough knowledge of accounting standards and auditing standards to execute the judgment. It is said that their judgments don't fluctuate compared with the novice, but a few cases reported that their judgments are completely opposite.

Total Risk: Auditors are required to evaluate the risks of the client companies. The risks are composed of inherent risk, control risk, detection risk, and business risk. Those risks by auditors are subjectively evaluated. We divide the evaluation as volatility and uncertainty.

Volatility: Volatility is defined as the fluctuations of the magnitude between the former condition and the present condition of the client companies. For example, a company's former profit is 100 and its present profit is 110, then the volatility of the profit of this company is $(110-100) / 100 = 0.1$. External auditor needs to evaluate the volatility of a client company.

Uncertainty: Uncertainty is defined as the fluctuations of the auditor's judgment. Because the auditors can't gather all the evidences about the client companies, they have something unknown to a greater or lesser extent. This is the cause of uncertainty.

Distribution: Volatility, uncertainty, risks related to auditors can be represented probabilities. Probability is a concept of statistics. Probability of event occurrence will be varied from 0 to 100. Each event has its own shape. Normal distribution is one of the typical event occurrence. If we know the shape of distribution, we can infer the result with probability.

Fluctuation of Audit Judgment: Auditor judgment will fluctuate along with the audit procedure. At the beginning of the audit engagement, as the auditors have a limited evidence, their conclusion is tentative, but at the end of the engagement, they have accumulated much more evidence than the beginning. As a result, their judgment will be fluctuated.

Dual Audit Procedure: Key audit points for example the accounts receivables are audited several times in an engagement. Such dual, triple audit procedures are required by audit standards. The fluctuations of auditor's judgment can be controlled by such dual audit procedure.

Audit Standards: Audit Standards are defined as guidelines for conducting an audit procedure. Auditors are required to comply with the Audit Standards. In old times (1950s), the Standards were very simple, but now the professional organizations such as American Institute of Certified Public Accountants are making detailed Audit Standards.

Compliance: Professional auditors are required to comply with Audit Standards. When the auditors could not detect accounting frauds, the professional organization's disciplinary committee investigates whether or not they were complied with Audit Standards.

Internal Control: After Enron scandal on 2001, Sarbanes and Oxley act enacted and required the listed companies to install internal control, and it was to be audited by the auditors in the United States. Almost all the countries followed the US. Internal Control was defined by a famous report called Committee of Sponsoring Organizations of Treadway Committee (COSO) in 1992.